ture diffusivity of heatproof materials in a wide range of temperatures and pressures of the gaseous medium indicate the correctness of the proposed mechanism.

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A METHOD OF MEASURING HEAT CONDUCTION IN A QUASISTEADY MODE WITH ASYMMETRIC BOUNDARY CONDITIONS AND WITH ALLOWANCE FOR NONLINEARITY

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The allowance for the dependence of the coefficient of thermal conductivity on the temperature and the accuracy of maintenance of the boundary condition in the absolute method of measurement of heat conduction in a quasisteady mode with asymmetric coundary conditions is analyzed by the methods of perturbation theory (iteration method).

The use of the relationships of a quasisteady mode with asymmetric boundary conditions [1] to determine the coefficient of thermal conductivity was evidently done most successively by Kaganer in measuring the properties of vacuum-shield insulation [2]. In his method a linear (or close to it) temperature rise was created at one boundary of the test specimen,

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while at the other boundary the heat flux was measured from the amount of cryogenic liquid evaporated. The coefficient of thermal conductivity was found as the ratio of the time derivative of the flux at the cold boundary to the rate of temperature rise at the hot boundary multiplied by the thickness of the specimen (absolute method of measurement).

In methods based on the relationships of a quasisteady mode with asymmetric boundary conditions it is particularly essential to allow for the dependence of the thermophysical parameters on the temperature, since the temperature difference over the thickness of the specimen may reach hundreds or even thousands of degrees by the end of the experiment. In this connection the introduction of corrections to the calculating equation and the determination of the allowable specimen thicknesses, rate of temperature rise, etc., are necessary in the development of methods. In the general formulation this problem is extremely complicated.

The allowance for the variation of the coefficient of thermal conductivity in the absolute method of measurement with a constant heat capacity is analyzed in the present report. Such a task is particularly necessary when studying the properties of vacuum-shield insulation, the heat capacity of which varies slightly while the effective thermal conductivity due to internal radiation can vary by an order of magnitude. At the same time we consider the allowance for nonlinearity in the maintenance of the boundary condition.

For the analysis of this problem we first consider the solution of the nonlinear equation of heat conduction with a constant heat capacity and a linear temperature dependence of the coefficient of thermal conductivity. In dimensionless form the equation of heat conduction and the boundary conditions have the form

$$\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial T}{\partial X}\left[(1+bT)\frac{\partial T}{\partial X}\right]; \tag{1}$$

$$\mathrm{Fo} = \frac{\lambda_0 \tau}{h^2 c\rho}; \quad \lambda = \lambda_0(1+bT); \quad X = \frac{x}{h};$$
$$T = 0 \quad X = 0; \quad T = f(\mathrm{Fo})\; X = 1. \tag{2}$$

We will seek the solution of (1), (2) by the iteration method. As the first step we integrate Eq. (1) without the left-hand term; i.e., we find the steady solution corresponding to the conditions (2):

$$(1+bT_1)\frac{\partial T_1}{\partial X} = C_1;$$

$$T_1 + \frac{bT_1^2}{2} = C_1 X + C_2; \quad C_2 = 0; \quad C_1 = f + \frac{bf^2}{2}; \tag{3}$$

$$T_1 + \frac{bT_1^2}{2} = \left(f + \frac{bf^2}{2}\right)X.$$

We note in passing that the constant C_1 when multiplied by λ_0/h is the heat flux through the system in the steady state. Solving the quadratic equation for T_1 and substituting it into (1), we find the expression for the temperature field in the second step:

$$T_2 = -\frac{1}{b} + \left\{\frac{1}{b^2} + \frac{2}{b}\left[X\left(f + \frac{bf^2}{2}\right) + \frac{f' + bff'}{3b^2\left(f + \frac{bf^2}{2}\right)} \times \right.\right.$$
$$\times\left(1 + \sqrt{1 + 2bX\left(f + \frac{bf^2}{2}\right)}\right)\left[b\left(f^2 + \frac{bf^2}{2}\right)X - 1\right] -$$
$$\left.\left. - \frac{b^2 f^2 X}{2}(3 + bf)\right]\right\}^{1/2}. \tag{4}$$

A solution by the iteration method is valid for small values of the criterion of temperature instability, i.e., when the following relation occurs:

$$\varepsilon = \frac{f' h^2 c\gamma}{f\lambda (1 + bt)} \ll 1. \quad (5)$$

A characteristic feature of the absolute method of measurement of the coefficient of thermal conductivity under consideration is the fact that the calculating equation includes, among the other parameters, only one quantity determined by the dependence $\lambda = \lambda(T)$, namely, $dq(0)/dFo$ at $X = 0$. We differentiate Eq. (4) with respect to the coordinate and time and find the time derivative of the flux at the cold boundary:

$$\frac{dq(0)}{d\text{Fo}} = \frac{\lambda_0}{h}\left\{f'(1 + bf) - \left[\frac{bf'^2}{6\left(1 + \frac{bf}{2}\right)^3} + \frac{(3 + bf) f''(1 + bf)}{6\left(1 + \frac{bf^2}{2}\right)^2}\right]\right\}. \quad (6)$$

An analysis of (6) shows that in the approximation under consideration the total expression for the derivative of the flux can be divided into three parts. The first of them corresponds to the derivative for the steady-state expression for the flux if in place of the constant temperatures at the boundary one substitutes a given variable value. This expression is the time derivative of the so-called transit flux (see [1]) but with allowance for the temperature dependence of the coefficient of thermal conductivity. The second and third parts characterize the effect of the unsteadiness of the process, also with allowance for the temperature dependence of the thermal conductivity, with the third part, characterizing the effect of the nonlinearity of the boundary condition, being reduced to zero for a linear time dependence of the temperature at the boundary.

An analysis shows that the fulfillment of the condition (5) leads to the fact that the second and third terms can be neglected.

The form of representation of Eq. (6) also does not contradict the physical meaning. In fact, if one imagines that some system with a variable coefficient of thermal conductivity is in a steady state in the presence of a temperature gradient and then one imposes on it a small perturbation — a slight change in temperature at one boundary with a constant temperature maintained at the other — then this small perturbation also makes a small contribution to the initial temperature field and flux field, i.e., to the steady-state fields with allowance for the temperature dependence of the coefficient of thermal conductivity.

A linear temperature dependence of the coefficient of thermal conductivity is inadequate for the analysis of real systems, but the use of the iteration method for the solution of a nonlinear equation of heat conduction even for a quadratic representation of $\lambda = \lambda(T)$ encounters serious mathematical difficulties: Already in the first step in the determination of the temperature T_1 it is necessary to solve a cubic equation. However, the analysis conducted above allows one to avoid these difficulties. In fact, since the contribution to the time derivative of the flux at the cold boundary is insignificant when the condition (5) is observed and it is neglected in the development of the method, we will consider only the expression for the transit flux with allowance for the temperature dependence of the coefficient of thermal conductivity. Its expression for quadratic, cubic, and other representations of $\lambda = \lambda(T)$ can easily be obtained from (3). Then after the dependence $dq(0)/dFo$ is obtained with a given condition $T = f(Fo)$ at the boundary the analysis of the experiment comes down to the choice of the required number of experimental values from the curves obtained and treatment by the method presented in [3], i.e, the solution of the system of equations for the determination of the coefficients as a function of $\lambda = \lambda(T)$.

An analysis of various specimens of materials and of experiments shows that the conditions of smallness of the parameter ε is not a rigid condition.

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